Chapter – 8

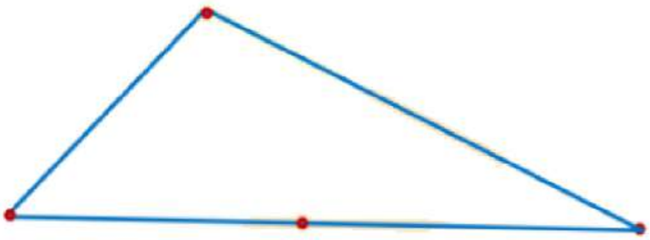
Quadrilaterals

Introduction to Quadrilaterals

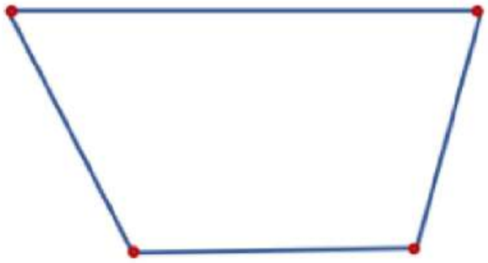
There are different shapes we can obtain by joining 4 points in different ways. Some shapes that we obtain are given below.

• When all 4 points are collinear (they lie on the same line). When we join them, we obtain a line segment.

• When 3 points are collinear. When we join the points in the following manner, we get a triangle.

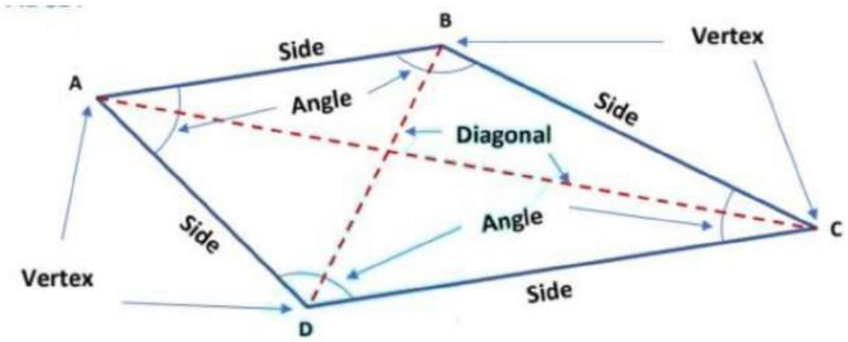


• No three points out of four are collinear, we obtain a closed figure with four sides. Such a figure formed by joining four points in order is called a quadrilateral.



A closed, two-dimensional figure formed by four-line segments is called a quadrilateral.

Thus, a plane figure bounded by four-line segments AB, BC, CD, and DA is called a quadrilateral ABCD and is written as quad. ABCD or ◻ ABCD.

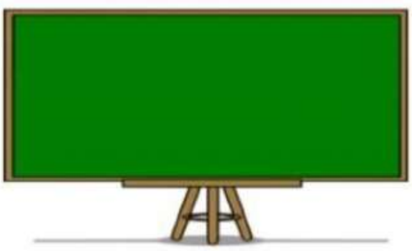


The word quadrilateral has originated from two Latin words, quad which means "four" and, lateral meaning "side".

A quadrilateral has four sides (AB, BC, CD, & DA), four angles (∠ ABC, ∠ BCD, ∠ CDA & ∠ DAB), four vertices(A, B, C & D) and two diagonals (AC & BD).

When we look around our surroundings, we find so many objects which are in the shape of a quadrilateral. For example, windows on the wall, the blackboard in your school, the top of the study table, the screen of the computer, the screen of the LCD TV, the screen of the mobile phone, pages in the book, etc. Some of these shapes are given below.

Window



LCD TV

Table

Computer

Terms related to Quadrilaterals

Let ABCD is a quadrilateral, which is shown in the following figure.

Adjacent sides: Two sides of a quadrilateral are consecutive or adjacent sides if they have a common point (vertex).

AB & BC are adjacent sides with common vertex B; BC & CD are adjacent sides with common vertex C; CD & DA are adjacent sides with common vertex D and DA & AB are adjacent sides with common vertex A.

Adjacent Side	Common Vertex
AB & BC	B
BC & CD	C
CD & DA	D
DA & AB	A

Opposite Sides: Two sides of a quadrilateral are opposite if they have no common endpoint (vertex).

In quadrilateral ABCD, AB & DC are opposite sides. Similarly, BC & AD are also a pair of opposite sides.

Adjacent Angles: Two angles are adjacent; if they have a common arm.

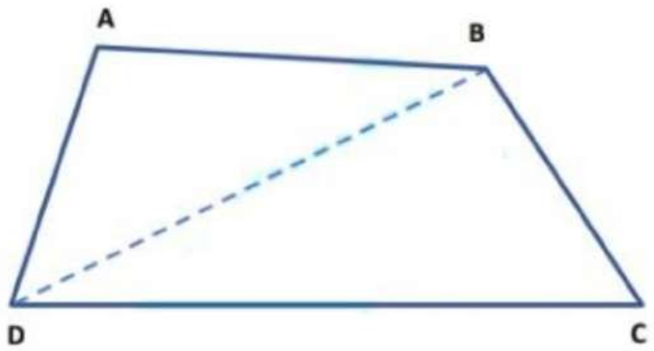
∠ ABC & ∠ BCD are adjacent angles with common arm BC; ∠ BCD & ∠ CDA are adjacent angles with common arm CD; ∠ CDA & ∠ DAB are adjacent angles with common arm DA and ∠ DAB & ∠ ABC are adjacent angles with common arm AB.

Adjacent angle	Common arm
∠ ABC & ∠ BCD	BC
∠ BCD & ∠ CDA	CD
∠ CDA & ∠ DAB	DA
∠ DAB & ∠ ABC	AB

Opposite Angles: Two angles of a quadrilateral are said to be opposite angles if they do not have a common arm. ∠ ABC & ∠ CDA; ∠ BCD & ∠ DAB are two pairs of opposite angles of □ ABCD.

Angle Sum Property of a Quadrilateral

Theorem 1: The sum of the four angles of a quadrilateral is 360°.



Given: ABCD is a quadrilateral

To Prove: $\angle ABC + \angle BCD + \angle CDA + \angle DAB = 360°$.

Construction: Join BD.

Proof: In Δ ABD, we have

Since the sum of all angles of a triangle is 180°.

$\angle DAB + \angle ABD + \angle BDA = 180°$. .... (I)

Similarly, in Δ CBD, we have

$\angle DBC + \angle BCD + \angle CDB = 180°$. .... (II)

Adding equation (I) & (II), we get

$\angle DAB + \angle ABD + \angle BDA + \angle DBC + \angle BCD + \angle CDB = 180° + 180°$

$\Longrightarrow \angle DAB + (\angle ABD + \angle DBC) + \angle BCD + (\angle CDB + \angle BDA) = 360°.$

$[\because \angle ABD + \angle DBC = \angle ABC \ \& \ \angle CDB + \angle BDA = \angle CDA]$

$\Longrightarrow \angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°.$

Example: In a quadrilateral ABCD, the angles A, B, C, and D are in the ratio 2: 3: 1: 4. Find the measure of each angle of the quadrilateral.

Solution: Since, angles A, B, C and D are in the ratio 2:3:1:4. For exact values, they contain a common factor x, which we have to find.

Let $\angle DAB = 2x°$, $\angle ABC = 3x°$, $\angle BCD = x°$ and $\angle CDA = 4x°$.

By the angle sum property of quadrilateral,

$\therefore \angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°.$

$\Longrightarrow 2x° + 3x° + x° + 4x° = 360°.$

$\Longrightarrow 10x° = 360°.$

$\Longrightarrow x° = \frac{360}{10}°$

$\Longrightarrow x° = 36°.$

Thus, the angles are

$\angle DAB = 2x° = 2 \times 36° = 72°$

$\angle ABC = 3x° = 3 \times 36° = 108°$

$\angle BCD = x° = 36°.$

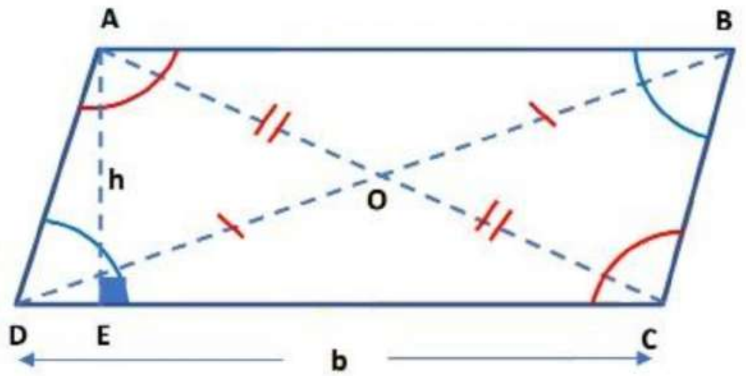
$\angle CDA = 4 \times 36° = 144°.$

Types of Quadrilaterals

Types of Quadrilaterals

Two Pairs of Parallel Lines	One Pair of Parallel Lines	Zero Pair of Parallel Lines
• Parallelogram • Rhombus • Rectangle • Square	• Trapezium • Isosceles Trapezium	• Kite

Parallelogram: A quadrilateral is a parallelogram if its pairs of opposite sides are parallel and equal in length.



Quadrilateral ABCD is a parallelogram because AB ∥ CD and BC ∥ AD.

• Diagonals bisect each other at O. i.e. AO = OC & DO = OB.

• Both pairs of opposite angles are equal. i.e. ∠ ADC = ∠ ABC & ∠ DAB = ∠ BCD.

• Consecutive angles are supplementary (Sum of these angles is 180°). i.e. ∠ ADC + ∠ BCD = 180° & ∠ DAB + ∠ ABC = 180°.

Area of Parallelogram

The area of a parallelogram is equal to the product of its base and its height.

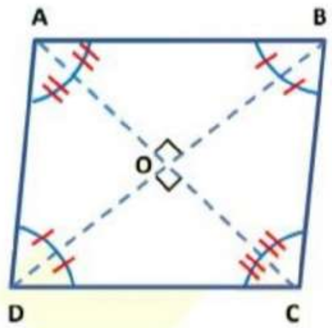
$$\text{Area} = \text{Base} \times \text{Height}$$

Area = b × h.

Note: Base is the side on which the height (perpendicular) is drawn.

Height is the perpendicular drawn from opposite vertex to its base.

Rhombus: A parallelogram having all sides equal is called a rhombus.



Thus, a parallelogram ABCD is a rhombus if AB = AD = BC = CD.

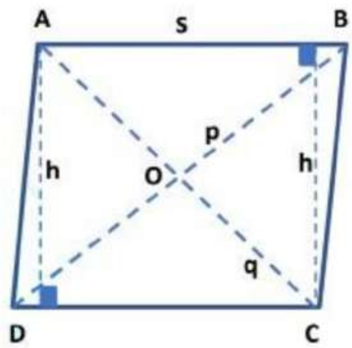
• Diagonals AC & DB are perpendicular to each other. i.e. $\angle AOB = 90°$, $\angle COD = 90°$, $\angle AOD = 90°$ and $\angle COB = 90°$,

• Diagonals AC bisects ∠DAB and ∠BCD and diagonal BD bisects ∠ABC and ∠ADC.

$\angle ABO = \angle CBO = \angle CDO = \angle ADO$ & $\angle BAO = \angle DAO = \angle BCO = \angle DCO$

Area of Rhombus

The Area of a rhombus is equal to the product of its side and its height.



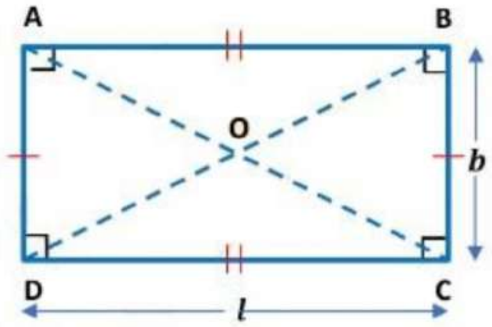
Area = side × height

Area = s × h.

Or, the area of a rhombus can also be found out by multiplying the lengths of the diagonals and then divide by 2.

Area = $\frac{pXq}{2}$; where p and q are diagonals of the rhombus.

Rectangle: A parallelogram in which each angle is a right angle and opposite sides are equal.



Thus, a parallelogram ABCD is a rectangle when AB = CD, BC = AD & ∠ ABC = ∠ BCD = ∠CDA = ∠ DAB = 90°.

In a rectangle, its diagonals are equal. i.e. AC = BD.

And diagonals bisect each other. So, OD = OB and OC = OA.

Area of a Rectangle

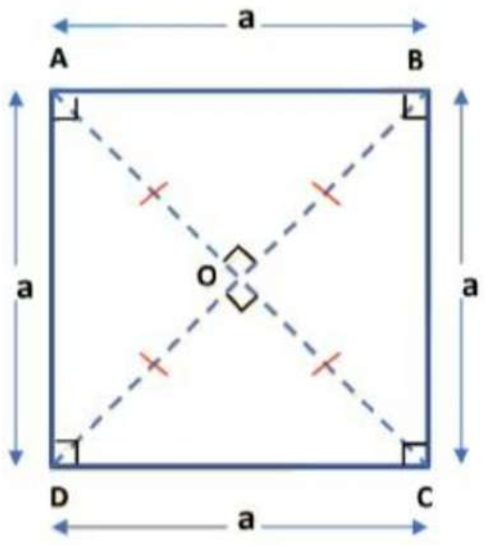
The area of the rectangle is equal to the product of its length and its breadth.

Area = length × breadth.

Area = l × b.

Square: A square is a rectangle with a pair of adjacent sides equal.

In other words, a parallelogram having all sides equal and each angle equal to a right angle is called a square.



Thus, a quadrilateral ABCD is a square in which AB = BC = CD = DA & ∠ ABC = ∠ BCD = ∠CDA = ∠ DAB = 90°.

A square has

- All the properties of a parallelogram.
- All the properties of a rectangle.
- All the properties of a rhombus.

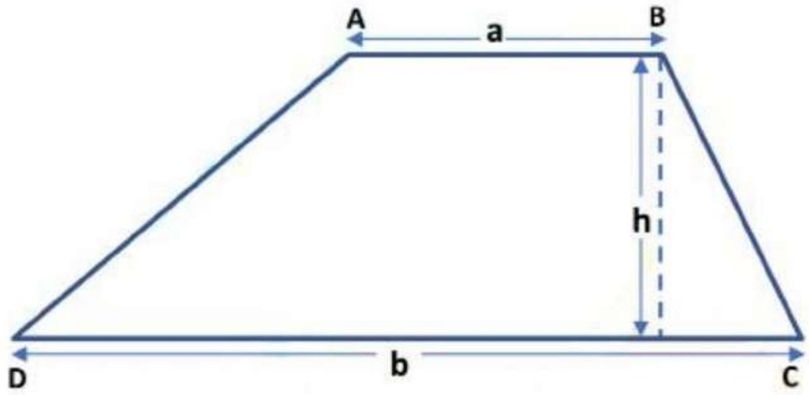
Area of a Square

The Area of square is equal to the square of its side length

Area = side × side

Area = a × a = a^2

Trapezium: A quadrilateral having exactly one pair of parallel sides is called a trapezium.



ABCD is a trapezium in which AB ∥ CD.

Area of Trapezium

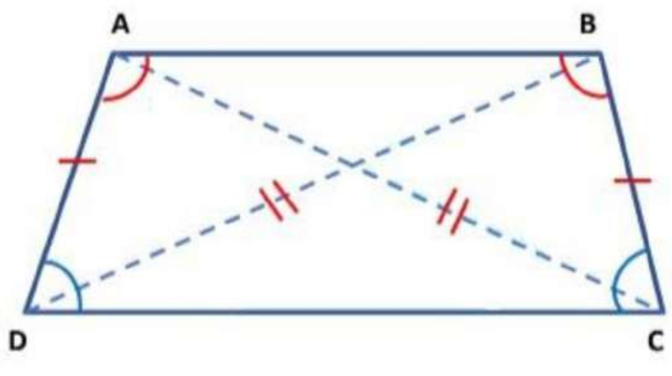
The area of trapezium is equal to the product of the sum of parallel sides and the distance between them.

The parallel sides are called the bases of the trapezium and the distance between the bases is the height of the trapezium.

Area $= \frac{1}{2} \times$ Sum of parallel sides $\times$ Distance between them.

Area $= \frac{1}{2} \times (a + b) \times h$.

Isosceles Trapezium: A trapezium is said to be an isosceles trapezium, if its non-parallel sides are equal.



Thus, a quadrilateral ABCD is isosceles trapezium in which AB ∥ CD and AD = BC.

In an isosceles trapezium

- Its diagonals AC and BD are equal.

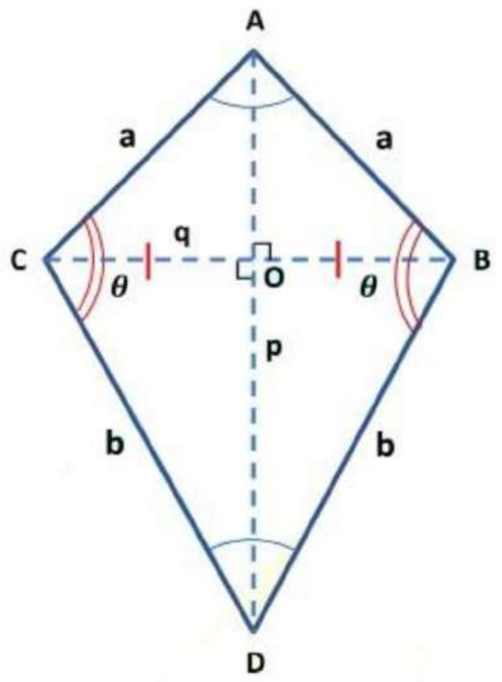
- Adjacent angles on non-parallel sides form linear pair of angles. So, ∠ ADC + ∠ DAB = 180° and ∠ BCD + ∠ ABC = 180°.

- Adjacent base angles are equal.

I. When DC is base then, ∠ ADC = ∠ BCD.

II. When AB is base then, ∠ DAB = ∠ ABC.

Kite: A quadrilateral is a kite if it has two pairs of equal adjacent sides and unequal opposite sides.



- Each pair is made of two adjacent sides that are equal in length.

So, AB = AC and DC = DB.

- The angles are equal where the pairs meet. So, ∠ ACD = ∠ ABD.

- The dotted lines are diagonals, which meet at a right angle.

So, ∠ AOB = ∠ COD = ∠ BOD = ∠ COA = 90°.

• Diagonal AD bisects BC. So, OC = OB.

Area of Kite

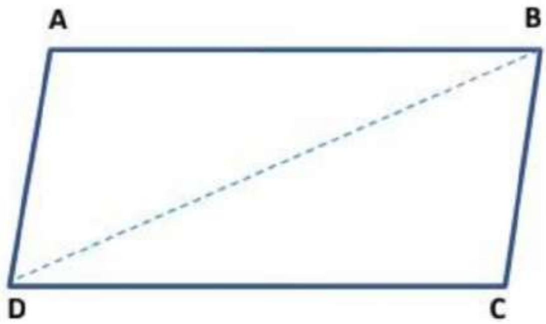
The area of a kite is half the product of lengths of its diagonals.

Area = $\frac{pXq}{2}$

Properties of Parallelogram

Properties of Parallelogram

Theorem 2: A diagonal of a parallelogram divides the parallelogram into two congruent triangles.



Given: ABCD is a parallelogram.

To prove: Δ ABD ≅ Δ CDB

Construction: Join BD.

Proof: Since ABCD is a parallelogram. Therefore, AB ∥ DC and BC ∥ AD.

Now, AB ∥ DC and transversal BD intersects them at B and D respectively.

∴ ∠ ABD = ∠ CDB .... (I) [Alternate interior angles]

Again, BC ∥ AD and transversal BD intersects them at B and D respectively.

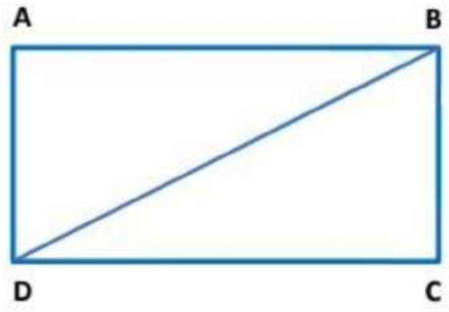
∴ ∠ ADB = ∠ DBC .... (II) [Alternate interior angles]

Now, in Δ ABD and Δ BDC, we have

∠ ABD = ∠ CDB	[From (I)]
BD = DB	[Common side]
∠ ADB = ∠ DBC	[From (II)]

Therefore, Δ ABD ≅ Δ CDB (By ASA-criterion of congruence)

Example: In the figure, quadrilateral ABCD is a rectangle in which BD is diagonal. Show that Δ ABD ≅ Δ CDB.



Given: ABCD is a rectangle in which BD is diagonal.

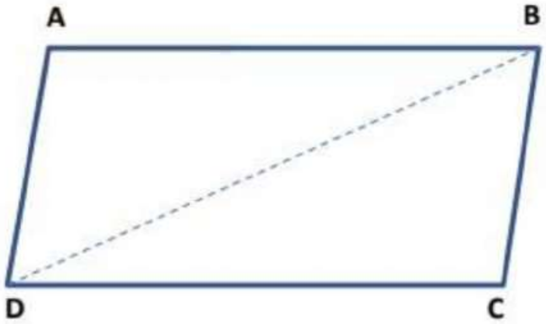
To prove: Δ ABD ≅ Δ CDB.

Proof: Quadrilateral ABCD is a rectangle. Therefore, ABCD is also a parallelogram.

Since a diagonal of a parallelogram divides it into two congruent triangles.

Hence, Δ ABD ≅ Δ CDB.

Theorem 3: In a parallelogram, opposite sides are equal.



Given: ABCD is a parallelogram.

To prove: AB = CD and AD = CB.

Construction: Join BD.

Proof: Since ABCD is a parallelogram. Therefore, AB ∥ DC and BC ∥ AD.

Now, AB ∥ DC and transversal BD intersects them at B and D respectively.

∴ $\angle ABD = \angle CDB$ .... (I) [Alternate interior angles]

Again, BC ∥ AD and transversal BD intersects them at B and D respectively.

∴ $\angle ADB = \angle DBC$ .... (II) [Alternate interior angles]

Now, in Δ ABD and Δ BDC, we have

$\angle ABD = \angle CDB$ [From (I)]

BD = DB [Common side]

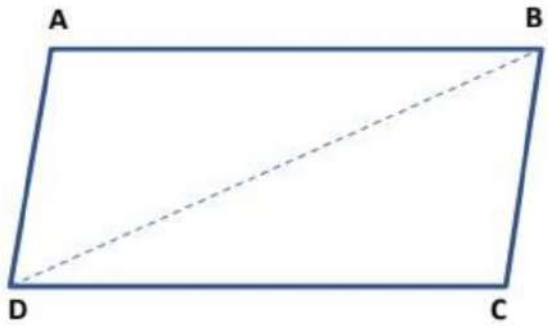
$\angle ADB = \angle DBC$ [From (II)]

Therefore, Δ ABD ≅ Δ CDB (By ASA-criterion of congruence)

By using corresponding parts of congruent triangles

⇒ AB = CD and AD = CB.

Theorem 4: If each pair of opposite sides of a quadrilateral is equal, then it is a parallelogram.



Given: A quadrilateral ABCD in which AB = CD and AD = BC

To prove: ABCD is a parallelogram.

Construction: Join BD.

Proof: Now, in Δ ABD and Δ CDB, we have

AB = CD [Given]

BD = DB [Common side]

AD = CB [Given]

Therefore, Δ ABD ≅ Δ CDB (By SSS-criterion of congruence)

By using corresponding parts of congruent triangles

⟹ ∠ ABD = ∠ CDB .... (I)

⟹ ∠ ADB = ∠ DBC .... (II)

Now, line BD intersects AB and CD at B and D, such that

∠ ABD = ∠ CDB [From (I)]

That is, alternate interior angles are equal.

⟹ AB ∥ CD .... (III)

Again, line BD intersects BC and AD at B and D such that.

∠ ADB = ∠ DBC [From (II)]

That is, alternate interior angles are equal.

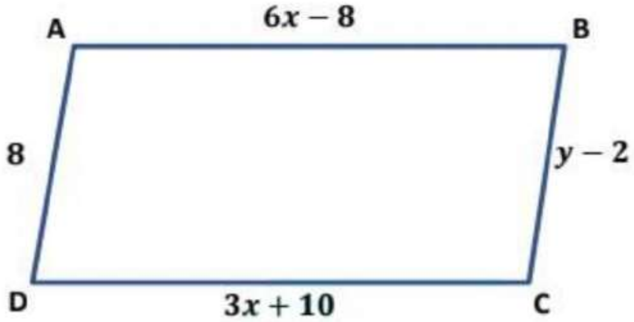
$\Rightarrow$ AB ∥ CD .... (IV)

From (III) and (IV), we have

AB ∥ CD and BC ∥ AD.

Hence, ABCD is a parallelogram.

Example: Find the values of x and y.



Solution: Quadrilateral ABCD is a parallelogram. So, opposite sides are equal. Then,

$$AB = DC$$

$\Rightarrow$ $6x - 8 = 3x + 10.$

$\Rightarrow$ $6x - 3x = 10 + 8.$

$\Rightarrow$ $3x = 18.$

$\Rightarrow$ $x = 18.3$

$\therefore$ $x = 6.$

Also, $BC = AD$

$\Rightarrow$ $y - 2 = 8$

$\Rightarrow$ $y = 8 + 2$

$\therefore$ $y = 10.$

Theorem 5: In a parallelogram, opposite angles are equal.

Given: ABCD is a parallelogram.

To prove: $\angle DAB = \angle BCD$ and $\angle ABC = \angle CDA$.

Proof: Since ABCD is a parallelogram. Therefore, AB ∥ CD and BC ∥ AD.

Now, AB ∥ CD and transversal AD intersects them at A and D respectively.

Since the sum of interior angles on the same side of the transversal is 180°.

$\therefore \quad \angle DAB + \angle CDA = 180° \quad \text{...... (I)}$

Similarly, BC ∥ AD and transversal CD intersect them at C and D respectively.

$\therefore \quad \angle CDA + \angle BCD = 180° \quad \text{...... (II)}$

From (I) and (II), we get

$$\angle DAB + \angle CDA = \angle CDA + \angle BCD$$

$\Rightarrow \quad \angle DAB = \angle BCD.$

Now, AD ∥ BC and transversal AB intersect them at A and B respectively.

Since the sum of interior angles on the same side of the transversal is 180°.

$\therefore \quad \angle DAB + \angle ABC = 180° \quad \text{...... (III)}$

Similarly, AB ∥ CD and transversal AD intersects them at A and D respectively.

$\therefore \quad \angle DAB + \angle CDA = 180° \quad \text{...... (IV)}$

From (III) and (IV), we get

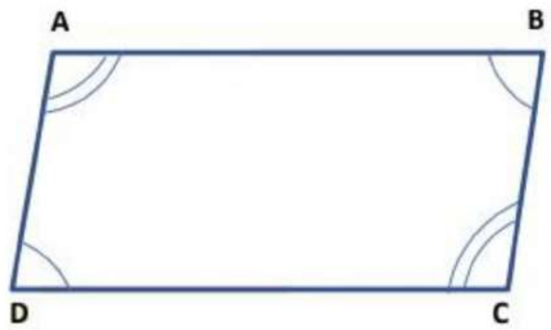
∠ DAB + ∠ ABC = ∠ DAB + ∠ CDA

⟹ ∠ ABC = ∠ CDA.

Similarly, ∠ ABC = ∠ CDA.

Hence, ∠ DAB = ∠ BCD and ∠ ABC = ∠CDA.

Theorem 6: If in a quadrilateral, each pair of opposite angles is equal, then it is a parallelogram.



Given: A quadrilateral ABCD in which ∠ DAB = ∠ BCD and ∠ ABC = ∠CDA.

To prove: ABCD is a parallelogram.

Proof: In quadrilateral ABCD, we have

∠ DAB = ∠ BCD (Given) .... (I)

⟹ ∠ ABC = ∠CDA (Given) .... (II)

From (I) & (II), we get

∠ DAB + ∠ ABC = ∠ BCD + ∠CDA .... (III)

Since, sum of the all interior angles of a quadrilateral is 360°

∴ ∠ DAB + ∠ ABC + ∠ BCD + ∠ CDA = 360° .... (IV)

⟹ ∠ DAB + ∠ ABC +∠ DAB + ∠ ABC = 360° [From (III)]

⟹ 2(∠ DAB + ∠ ABC) = 360°

⟹ ∠ DAB + ∠ ABC = 180°

∴ $\angle DAB + \angle ABC = \angle BCD + \angle CDA = 180°$ .... (V)

Now, line AB intersects AD and BC at A and B respectively, such that

$\angle DAB + \angle ABC = 180°$ [From (V)]

That is, the sum of interior angles on the same side of the transversal is 180°.

⟹ $AD \parallel BC$ .... (VI)

Again, line BC intersects AB and CD at B and C respectively, such that.

$\angle BCD + \angle ABC = 180°$ [∵ $\angle DAB = \angle BCD$]

That is, the sum of interior angles on the same side of the transversal is 180°.

⟹ $AB \parallel CD$ .... (VII)

From (VI) and (VII), we have

$AB \parallel CD$ and $AD \parallel BC$.

Hence, ABCD is a parallelogram.

Example: A parallelogram ABCD is shown in the figure. Find the missing angles.

Given: $\angle DAB = 65°$.

To find: $\angle ABC$, $\angle BCD$, and $\angle CDA$.

Solution: ABCD is a parallelogram. So, the opposite angles are equal.

$\angle BCD = \angle DAB$.

$\angle BCD = 65°$. .... (I)

And also, adjacent angles are supplementary. We have

$\angle BCD + \angle ABC = 180°$

$\Rightarrow$ $65° + \angle ABC = 180°$ [From (I)]

$\Rightarrow$ $\angle ABC = 180° - 65°$

$\Rightarrow$ $\angle ABC = 115°$. .... (II)

Since, in a parallelogram, opposite angles are equal.

Therefore, $\angle CDA = \angle ABC$.

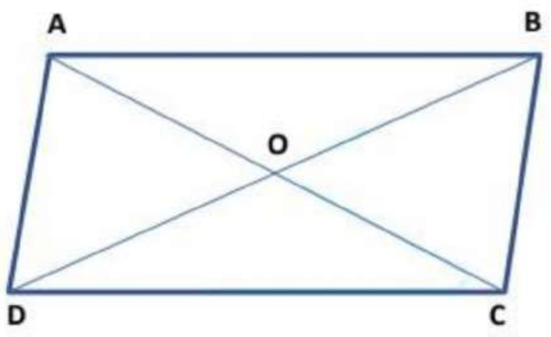
$\Rightarrow$ $\angle CDA = 115°$. [From (II)]

Hence, $\angle ABC = 115°$.

$\angle BCD = 65°$.

And, $\angle CDA = 115°$.

Theorem 7: The diagonals of a parallelogram bisect each other.



Given: ABCD is a parallelogram.

To prove: OA = OC and OD = OB.

Proof: Since ABCD is a parallelogram. Therefore, AB ∥ CD and BC ∥ AD.

Now, AB ∥ CD and transversal BD intersects them at B and D respectively.

∴ $\angle ABD = \angle CDB$ [Alternate interior angles]

⟹ $\angle ABO = \angle CDO$ .... (I)

Again, AB ∥ CD and transversal AC intersects them at A and C respectively.

∴ $\angle BAC = \angle DCA$ [Alternate interior angles]

⟹ $\angle BAO = \angle DCO$ .... (I)

Now, in Δ AOB and Δ COD, we have

$\angle ABO = \angle CDO$ [From (I)]

AB = CD [Opposite side of parallelogram are equal]

$\angle BAO = \angle DCO$ [From (II)]

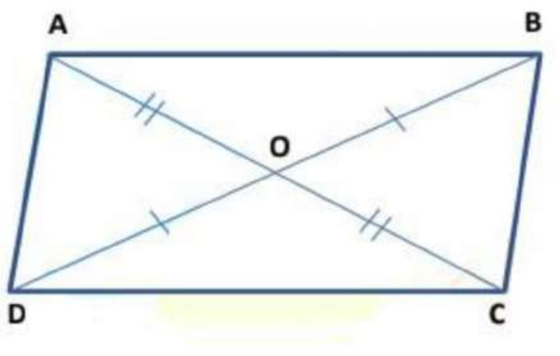
Therefore, Δ AOB ≅ Δ COD (By ASA-criterion of congruence)

By using corresponding parts of congruent triangles

⇒ OA = OC and OD = OB.

Hence, the diagonals of a parallelogram bisect each other.

Theorem 8: If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.



Given: A quadrilateral ABCD in which the diagonals AC and BD intersect at O such that OA = OC and OD = OB.

To prove: AB ∥ CD & AD ∥ BC

Proof: In Δ AOD and Δ COB, we have

$OA = OC$ (Given)

$\angle AOD = \angle COB$ (Vertically opposite angles)

$OD = OB$ (Given)

Therefore, $\Delta AOD \cong \Delta COB$ (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle OAD = \angle OCB$ .... (I)

Now, line AC intersects BC and AD at C and A respectively, such that

$\angle OAD = \angle OCB$ [From (I)].

That is, alternate interior angles are equal.

$\therefore$ $AD \parallel BC$. .... (II)

Now, in Δ AOB and Δ COD, we have

$OB = OD$ (Given)

$\angle AOB = \angle COD$ (Vertically opposite angles)

$OA = OC$ (Given)

Therefore, $\Delta AOB \cong \Delta COD$ (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $\angle OBA = \angle ODC$ .... (III)

Now, line BD intersects AB and DC at B and D respectively, such

that $\angle OBA = \angle ODC$ [From (III)].

That is, alternate interior angles are equal.

$\therefore$ $AB \parallel DC$. .... (IV)

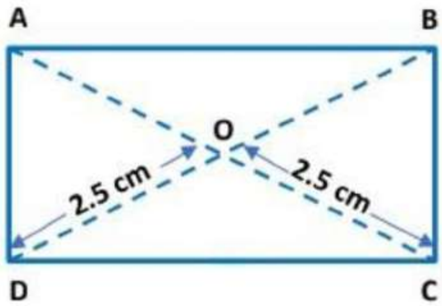
From equation (II) & (IV), we get

Hence, ABCD is a parallelogram.

Example: Find the length of the following diagonals in the parallelogram ABCD:

I. AC

II. BD



Given: $OD = OC = 2.5$ cm.

To find: Length of AC & BD.

Solution: ABCD is a parallelogram. So, the diagonals of parallelogram ABCD bisect each other. Then, BD bisects the AC.

$\Rightarrow$ $OA = OC$

$OA = 2.5$ cm.

Then, $AC = OA + OC$

$\Rightarrow$ $AC = 2.5 \text{ cm} + 2.5 \text{ cm}$.

$\Rightarrow$ $AC = 5$ cm.

And, AC bisects the BD. Then,

$\Rightarrow$ $OD = OB$

$OD = 2.5$ cm.

Then, $BD = OD + OB.$

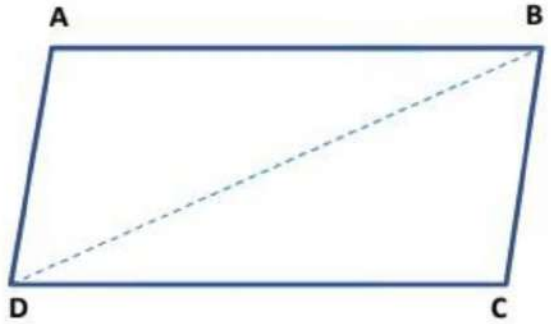
$\Rightarrow$ $BD = 2.5\text{ cm} + 2.5\text{ cm}.$

$\Rightarrow$ $BD = 5\text{ cm}.$

Another Condition for a Quadrilateral to be a Parallelogram

Another Condition for a Quadrilateral to be a Parallelogram

Theorem 9: A quadrilateral is a parallelogram if a pair of the opposite side is equal and parallel.



Given: A quadrilateral ABCD in which $AB = CD$ and, $AB \parallel CD$.

To prove: Quadrilateral ABCD is a parallelogram.

Construction: Join BD.

Proof: Now, in Δ BAD and Δ DCB, we have

$AB = CD$ (Given)

Since $AB \parallel CD$ and transversal BD intersects at B and D, so alternate interior angles are equal.

$\Longrightarrow$ $\angle CDB = \angle ABD$

$BD = DB$ (Common)

Therefore, $\Delta BAD \cong \Delta DCB$ (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow \qquad \angle ADB = \angle CBD$

Now, line BD intersects AB and DC at B and D respectively, such that $\angle ADB = \angle CBD$

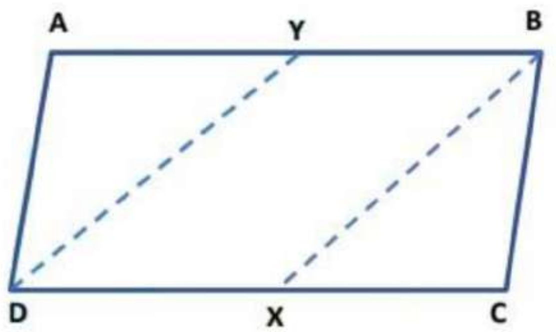
That is, alternate interior angles are equal.

$\therefore$ AD ∥ BC.

Thus, AB ∥ CD and AD ∥ BC.

Hence, quadrilateral ABCD is a parallelogram.

Example: In the figure, ABCD is a parallelogram and X, Y are the mid- points of sides AB and DC respectively. Show that quadrilateral DXBY is a parallelogram.



Given: ABCD is a parallelogram in which X and Y are the mid-points of AB and DC respectively.

To prove: Quadrilateral DXBY is a parallelogram.

Construction: Join DX and BX.

Proof: Since X and Y are the mid-points of DC and AB respectively.

$\therefore \qquad YB = \frac{1}{2} AB$ and $DX = \frac{1}{2} DC. \qquad$ .... (I)

But, $\qquad AB = DC \qquad$ [$\because$ ABCD is a parallelogram]

$\Rightarrow \qquad \frac{1}{2} AB = \frac{1}{2} DC$

⟹ YB = DX. [From(I)] .... (II)

Also, AB ∥ DC [∵ ABCD is a parallelogram]

∴ YB ∥ DX .... (III)

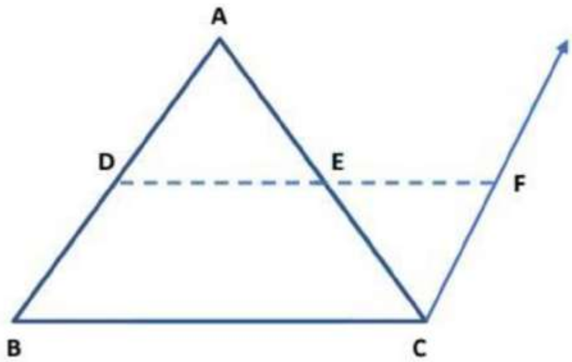
Since a quadrilateral is a parallelogram if a pair of the opposite side is equal and parallel.

From (II) & (III), we get Quadrilateral DXBY is a parallelogram.

The Mid-point Theorem

The Mid-point Theorem

Theorem 10: The line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of it.



Given: A Δ ABC in which D and E are the mid-points of sides AB and AC respectively.

To prove: DE ∥ BC and DE = $\frac{1}{2}$ BC.

Construction: Produce the line segment DE to F, such that DE = EF.

Join FC.

Proof: Now, in Δ AED and Δ CEF, we have

AE = CE (∵ E is the mid-point of AC)

$\Rightarrow$ $\angle AED = \angle CEF$ (Vertically opposite angles)

And, $DE = FE$ (By construction)

Therefore, $\Delta AED \cong \Delta CEF$ (By SAS-criterion of congruence)

By using corresponding parts of congruent triangles

$\Rightarrow$ $AD = CF$ .... (I)

And, $\angle ADE = \angle CFE$ .... (II)

Now, D is the mid-point of AB.

$\Rightarrow$ $AD = DB$ .... (III)

From equation (I) & (III), we get

$CF = DB$ .... (IV)

Now, DF intersects AD and FC at D and F respectively such that

$\angle ADE = \angle CFE$ [From (II)]

That is, alternate interior angles are equal.

$\therefore$ $AD \parallel FC$.

$\Rightarrow$ $DB \parallel CF$. .... (V)

From (III) & (V), we find that DBCF is a quadrilateral such that one pair of sides is equal and parallel.

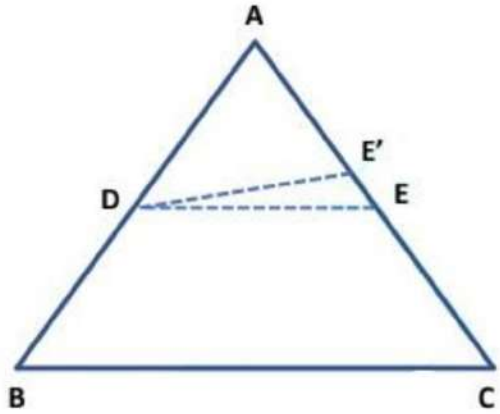
$\therefore$ DBCF is a parallelogram.

$\Rightarrow DF \parallel BC$ and $DF = BC$ ($\because$ Opposite sides of a parallelogram are equal and parallel)

But, D, E, F are collinear and $DE = EF$.

$\therefore$ $DE \parallel BC$ and $DE = \frac{1}{2} BC$.

Theorem 11: The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side.



Given: Δ ABC in which D is the mid-point of AB and DE ∥ BC

To prove: E is the mid-point of AC.

Construction: Join DE and DE'.

Proof: We have to prove that E is the mid-point of AC. Suppose, E is not the mid-point of AC. Then, let E' be the mid-point of AC.

Now, in Δ ABC, D is the mid-point of AB and E' is the mid-point of AC.

But the line drawn through the mid-point of one side of a triangle,

parallel to another side bisects the third side.

Therefore, we have

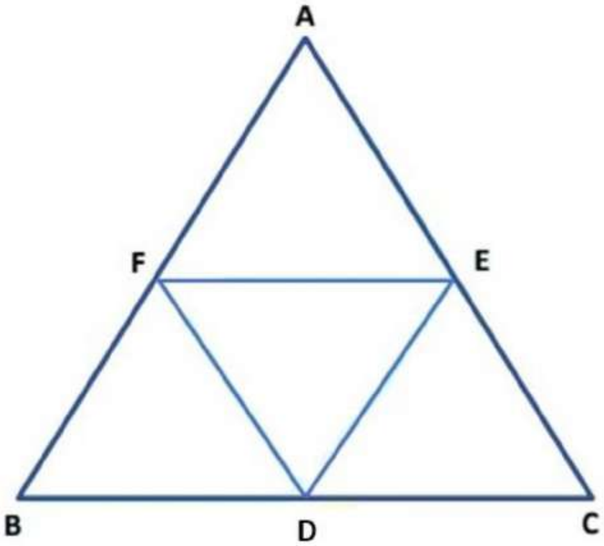
DE' ∥ BC. .... (I)

Also Given, DE ∥ BC. .... (II)

From (I) and (II), we find that two intersecting lines DE and DE' are both parallel to line BC. This is a contradiction.

So, our assumption is wrong. Hence, E is the mid-point of AC.

Example: In the figure, D, E, and F are respectively the mid-points of sides BC, CA, and AB of an equilateral triangle ABC. Prove that triangle DEF is also an equilateral triangle.



Given: D, E, and F are respectively the mid-points of sides BC, CA,

and AB of an equilateral triangle ABC.

To prove: DEF is an equilateral triangle.

Construction: Join DE, DF, and FE.

Proof: Since the segment joining the mid-points of two sides of a triangle is half of the third side. Therefore, D and E are the mid-points of BC and AC respectively.

$\Rightarrow$ $DE = \frac{1}{2} AB$ .... (I)

E and F are the mid-points of AC and AB respectively.

$\therefore$ $EF = \frac{1}{2} BC.$ .... (II)

F and D are the mid-points of AB and BC respectively.

$\therefore$ $FD = \frac{1}{2} AC.$ .... (III)

Now, Δ ABC is an equilateral triangle.

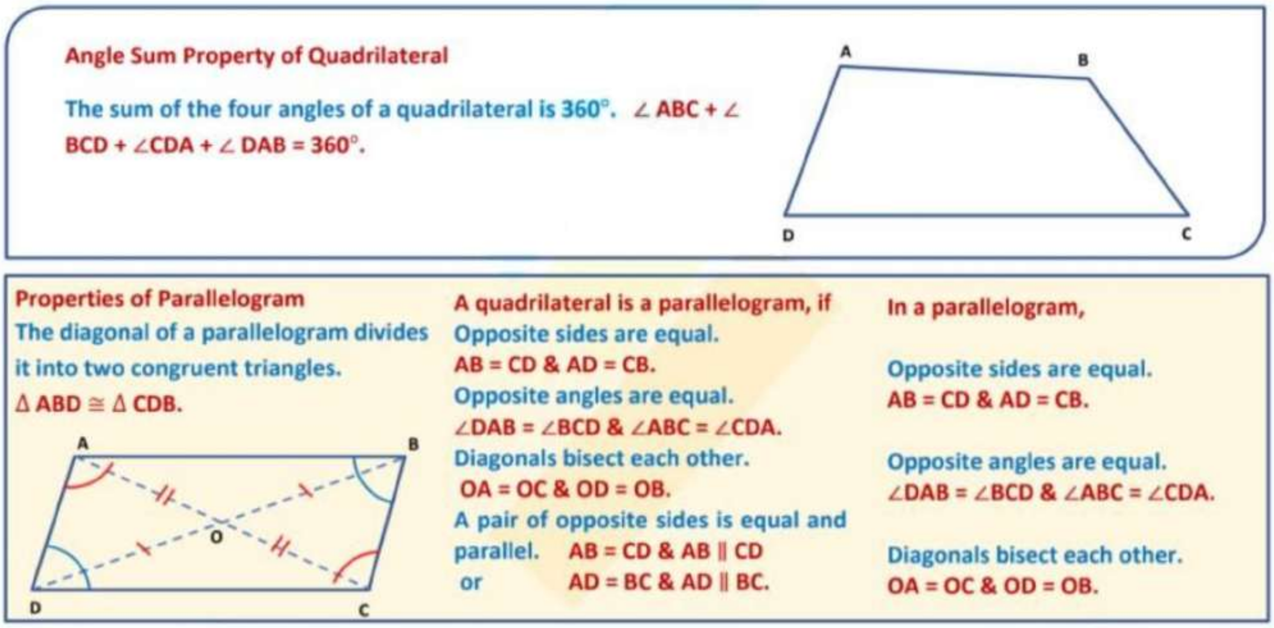
$$\Rightarrow \quad AB = BC = CA$$

$$\Rightarrow \quad \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$$

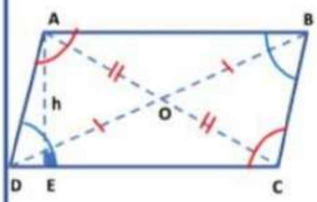
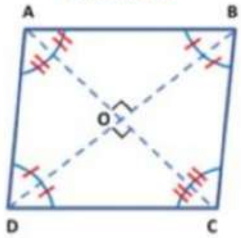
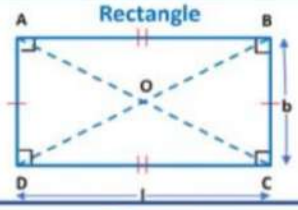
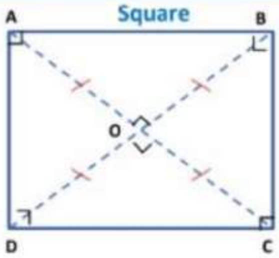
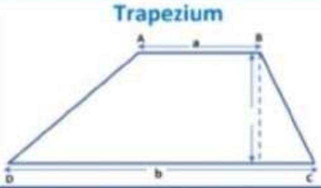
$$\Rightarrow \quad DE = EF = FD.$$

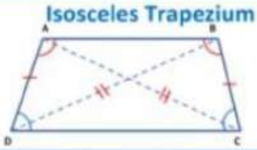
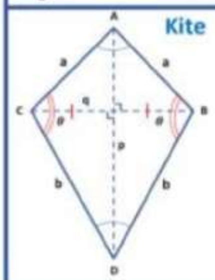
Hence, Δ DEF is an equilateral triangle.

Summary of Quadrilaterals



Types of parallelograms

Quadrilaterals	Sides	Angles	Diagonals
Parallelogram 	Pairs of opposite sides are parallel and equal in length.	Both pairs of opposite angles are equal. Consecutive angles are supplementary.	Diagonals bisect each other
Rhombus 	All sides are equal.	Both pairs of opposite angles are equal. Consecutive angles are supplementary.	Diagonals bisect each other at right angles.
Rectangle 	Pairs of opposite sides are parallel and equal in length.	Both pairs of opposite angles are equal. Consecutive angles are supplementary. Each angle is a right angle	Diagonals bisect each other and are equal
Square 	All sides are equal.	Both pairs of opposite angles are equal. Consecutive angles are supplementary. Each angle is equal to a right angle.	Diagonals bisect each other at right angles and are equal.
Trapezium 	Exactly one pair of sides are parallel.	-	-

Isosceles Trapezium 	Non-parallel sides are equal.	Adjacent angles on non-parallel sides form linear pair of angles. Adjacent base angles are equal.	Diagonals are equal.
Kite 	One pair of adjacent sides are equal as well as unequal.	The angles are equal where the pairs meet.	Diagonals meet at a right angle. One of sthe diagonal bisects the others.

The Mid-point Theorem

The line segment joining the mid-points of two sides of a triangle is parallel to the third side and equal to half of it. DE $\parallel$ BC & DE = $\frac{1}{2}$ BC.

The line drawn through the mid-point of one side of a triangle, parallel to another side bisects the third side. E is the mid-point of AC